ONLINE APPENDIX

(NOT FOR PUBLICATION)

A. Theory Appendix

A.1 Model Derivation

Derivation of optimal lobbying inputs and profits. I derive expressions for firms' optimal lobbying and profits conditional on lobbying. I first characterize non-exporters' optimal lobbying inputs and profits. Conditional on lobbying amounts of b, non-exporters' profits are

$$\pi^d(b;\psi) = \frac{1}{\sigma}\Big(\frac{\mu w}{\phi}\Big)^{1-\sigma}\tau^{\sigma} b^{\theta\sigma} P^{\sigma-1} E - w\Big(\kappa\frac{b}{\eta} + f_b + f\Big) = \tilde{\pi}^d(0;\psi) b^{\theta\sigma} - w\Big(\kappa\frac{b}{\eta} + f_b + f\Big) \tag{A.1}$$

where $\tilde{\pi}^d(0;\psi)$ are non-exporters' variable profits conditional on not lobbying.

Firms choose their optimal lobbying inputs that maximize profits, characterized by the first-order conditions (FOC). Taking the derivative with respect to b, I obtain the following FOC:

$$\kappa\frac{w}{\eta} = \theta\sigma\tilde{\pi}^d(0;\psi) b^{(\theta\sigma-1)}.$$

After arranging the above equation, the optimal lobbying inputs can be expressed as follows:

$$b^d = \Big(\frac{\theta\sigma\eta}{\kappa w}\tilde{\pi}^d(0;\psi)\Big)^{\frac{1}{1-\theta\sigma}}.$$

After substituting the optimal lobbying inputs into Equation (A.1), I obtain that

$$\pi^d(b;\psi) = (1-\theta\sigma)\Big(\frac{\theta\sigma\eta}{\kappa w}\Big)^{\frac{\theta\sigma}{1-\theta\sigma}} \tilde{\pi}^d(0;\psi)^{\frac{1}{1-\theta\sigma}} - w(f + f_b).$$

Exporters' optimal lobbying inputs and profits can be derived similarly. Conditional on lobbying inputs of b, exporters' profits are

$$\begin{aligned}\pi^x(b;\psi) &= \Big[\frac{1}{\sigma}\Big(\frac{\mu w}{\phi}\Big)^{1-\sigma}\tau^{\sigma} P^{\sigma-1} E + \frac{1}{\sigma}\Big(\frac{\mu \tau_x w}{\phi}\Big)^{1-\sigma}\tau^{\sigma} P_f^{\sigma-1} E_f\Big] b^{\theta\sigma} - w\Big(\kappa\frac{b}{\eta} + f_b + f + f_x\Big) \\ &= \big(\tilde{\pi}^x(0;\psi) + \tilde{\pi}^x(0;\psi)\big) b^{\theta\sigma} - w\Big(\kappa\frac{b}{\eta} + f_b + f + f_x\Big).\end{aligned}$$

where $\tilde{\pi}^x(0;\psi)$ are non-exporters' variable profits conditional on not lobbying. From the FOC with respect to b, the optimal lobbying inputs are expressed as

$$b^x = \Big(\frac{\theta\sigma\eta}{\kappa w}\big(\tilde{\pi}^x(0;\psi) + \tilde{\pi}^x(0;\psi)\big)\Big)^{\frac{1}{1-\theta\sigma}}.$$

After substituting the optimal lobbying inputs, I obtain that

$$\pi^x(b;\psi) = (1-\theta\sigma)\Big(\frac{\theta\sigma\eta}{\kappa w}\Big)^{\frac{\theta\sigma}{1-\theta\sigma}} \big(\tilde{\pi}^d(0;\psi) + \tilde{\pi}^x(0;\psi)\big)^{\frac{1}{1-\theta\sigma}} - w(f + f_b + f_x).$$

Zero profit cutoff. The zero profit cutoff productivity satisfies that $\pi(\bar{\phi}^e(\tau,\eta),\tau,\eta) = 0$, which is determined as

$$\bar{\phi}^e(\tau,\eta) = \left[\frac{\sigma f}{\frac{1}{\sigma}(\mu w)^{1-\sigma}P^{\sigma-1}E}\right]^{\frac{1}{\sigma-1}}. \tag{A.2}$$

Lobbying cutoff. The lobbying cutoff is characterized as

$$\max\Big\{\pi^d(0;\bar{\phi}^b(\tau,\eta),\tau,\eta), \pi^d(0;\bar{\phi}^b(\tau,\eta),\tau,\eta) + \pi^x(0;\bar{\phi}^b(\tau,\eta),\tau,\eta)\Big\}$$
$$= \max\Big\{\pi^d(b^d;\bar{\phi}^b(\tau,\eta),\tau,\eta), \pi^x(b^x;\bar{\phi}^b(\tau,\eta),\tau,\eta)\Big\},$$

where the left- and right-hand sides are the maximum profits conditional on not lobbying and lobbying, respectively.

More specifically, when η is sufficiently high, non-exporters may participate in lobbying, that is, $\bar{\phi}^b(\tau,\eta) < \bar{\phi}^x(\tau,\eta)$. In such a case, the lobbying cutoff is implicitly defined by the following condition:

$$(1-\theta\sigma)\Big(\frac{\theta\sigma\eta}{\kappa w}\Big)^{\frac{\theta\sigma}{1-\theta\sigma}}\Big(\frac{1}{\sigma}\Big(\frac{\mu w}{\bar{\phi}^b(\tau,\eta)}\Big)^{1-\sigma}\tau^\sigma P^{\sigma-1}E\Big)^{\frac{1}{1-\theta\sigma}} - wf_b = \frac{1}{\sigma}\Big(\frac{\mu w}{\bar{\phi}^b(\tau,\eta)}\Big)^{1-\sigma}\tau^\sigma P^{\sigma-1}E, \tag{A.3}$$

In the case in which $\bar{\phi}^b(\tau,\eta) \geq \bar{\phi}^x(\tau,\eta)$ holds, the lobbying cutoff is implicitly defined by the following condition:

$$(1-\theta\sigma)\Big(\frac{\theta\sigma\eta}{\kappa w}\Big)^{\frac{\theta\sigma}{1-\theta\sigma}}\Big(\frac{1}{\sigma}\Big(\frac{\mu w}{\bar{\phi}^b(\tau,\eta)}\Big)^{1-\sigma}\tau^\sigma(P^{\sigma-1}E + \tau_x^{1-\sigma}P_f^{\sigma-1}E_f)\Big)^{\frac{1}{1-\theta\sigma}} - wf_b$$
$$= \frac{1}{\sigma}\Big(\frac{\mu w}{\bar{\phi}^b(\tau,\eta)}\Big)^{1-\sigma}\tau^\sigma(P^{\sigma-1}E + \tau_x^{1-\sigma}P_f^{\sigma-1}E_f). \tag{A.4}$$

Export cutoff. The export cutoff is characterized by

$$\max\Big\{\pi^d(0;\bar{\phi}^x(\tau,\eta),\tau,\eta) + \pi^x(0;\bar{\phi}^x(\tau,\eta),\tau,\eta), \pi^x(b^x;\bar{\phi}^x(\tau,\eta),\tau,\eta)\Big\}$$
$$= \max\Big\{\pi^d(0;\bar{\phi}^x(\tau,\eta),\tau,\eta), \pi^d(b^d;\bar{\phi}^x(\tau,\eta),\tau,\eta)\Big\},$$

where the left-and right-hand sides are the maximum profits conditional on exporting and not exporting, respectively.

More specifically, in the case in which $\bar{\phi}^b(\tau,\eta) \geq \bar{\phi}^x(\tau,\eta)$ holds, the export cutoff satisfies that

$$\frac{1}{\sigma}\Big(\frac{\mu\tau_x w}{\bar{\phi}^x(\tau,\eta)}\Big)^{1-\sigma}\tau^\sigma P_f^{\sigma-1}E_f = wf_x.$$

From this condition, the export cutoff can be expressed as follows:

$$\bar{\phi}^x(\tau,\eta) = \left(\frac{wf_x}{\frac{1}{\sigma}(\mu\tau_x w)^{1-\sigma}\tau^\sigma P_f^{\sigma-1}E_f}\right)^{\frac{1}{\sigma-1}}. \tag{A.5}$$

In the case where $\bar{\phi}^b(\tau,\eta) < \bar{\phi}^x(\tau,\eta)$, the export cutoff satisfies

$$(1-\theta\sigma)\Big(\frac{\theta\sigma\eta}{\kappa w}\Big)^{\frac{\theta\sigma}{1-\theta\sigma}}\bigg(\frac{1}{\sigma}\Big(\frac{\mu w}{\bar{\phi}^x(\tau,\eta)}\Big)^{1-\sigma}\tau^\sigma(P^{\sigma-1}E+\tau_x^{1-\sigma}P_f^{\sigma-1}E_f)\bigg)^{\frac{1}{1-\theta\sigma}} - wf_x$$
$$= (1-\theta\sigma)\Big(\frac{\theta\sigma\eta}{\kappa w}\Big)^{\frac{\theta\sigma}{1-\theta\sigma}}\bigg(\frac{1}{\sigma}\Big(\frac{\mu w}{\bar{\phi}^x(\tau,\eta)}\Big)^{1-\sigma}\tau^\sigma P^{\sigma-1}E\bigg)^{\frac{1}{1-\theta\sigma}}.$$

From this condition, the export cutoff can be expressed as follows:

$$\bar{\phi}^x(\tau,\eta) = \left(\frac{\sigma^{\frac{1}{1-\theta\sigma}}wf_x}{(1-\theta\sigma)(\frac{\theta\sigma\eta}{\kappa w})^{\frac{\theta\sigma}{1-\theta\sigma}}(\mu w)^{\frac{1-\sigma}{1-\theta\sigma}}\tau^\sigma\big((P^{\sigma-1}E+\tau_x^{1-\sigma}P_f^{\sigma-1}E)^{\frac{1}{1-\theta\sigma}}-(P^{\sigma-1}E)^{\frac{1}{1-\theta\sigma}}\big)}\right)^{\frac{1-\theta\sigma}{\sigma-1}}. \qquad \text{(A.6)}$$

A.2 Derivation of Equation (2.8)

The total labor used for production can be written as follows:

$$L^p = M\bigg[\int\frac{q(\psi)}{\phi}d\hat{G}(\psi) + \int x(\psi)\frac{q^x(\psi)}{\phi}d\hat{G}(\psi)\bigg].$$

Dividing both sides by $Q = \Big[M\int q(\psi)^{\frac{\sigma-1}{\sigma}}d\hat{G}(\psi) + M_f\int x_f(\psi)q_f^x(\psi)^{\frac{\sigma-1}{\sigma}}d\hat{G}_f(\psi)\Big]^{\frac{\sigma}{\sigma-1}}$, I can obtain that

$$\frac{L^p}{Q} = M\bigg[\int\frac{1}{\phi}\frac{q(\psi)}{Q}d\hat{G}(\psi) + \int x(\psi)\frac{1}{\phi}\frac{q^x(\psi)}{Q}d\hat{G}(\psi)\bigg],$$

which can be expressed as follows:

$$\frac{L^p}{Q} = M^{-\frac{1}{\sigma-1}}\bigg[\int\frac{1}{\phi}\frac{q(\psi)}{\tilde{q}}d\hat{G}(\psi) + \int x(\psi)\frac{1}{\phi}\frac{q^x(\psi)}{\tilde{q}}d\hat{G}(\psi)\bigg],$$

where $\tilde{q}$ is defined as follows:

$$\tilde{q} = \bigg[\int q(\psi)^{\frac{\sigma-1}{\sigma}}d\hat{G}(\psi) + \frac{M_f}{M}\int x_f(\psi)q_f^x(\psi)^{\frac{\sigma-1}{\sigma}}d\hat{G}_f(\psi)\bigg]^{\frac{\sigma}{\sigma-1}}.$$

Rearranging the terms, I can rewrite Q as follows:

$$Q = AL, \quad \text{where} \quad A = M^{\frac{1}{\sigma-1}}\times\bigg[\int\frac{1}{\phi}\frac{q(\psi)}{\tilde{q}}d\hat{G}(\psi) + \int x(\psi)\frac{1}{\phi}\frac{q^x(\psi)}{\tilde{q}}d\hat{G}(\psi)\bigg]^{-1}\times\frac{L^p}{L}.$$

Because $\mathbb{W} = Q/L$, when $d\ln(L^p/L)$ is sufficiently small,

$$d\ln\mathbb{W} \approx \frac{1}{\sigma-1}d\ln M + d\ln\bigg[\int x(\psi)\frac{1}{\phi}\frac{q^x(\psi)}{\tilde{q}}d\hat{G}(\psi)\bigg]^{-1}.$$

Comparison between the allocative efficiency Terms in Equation (2.8) and Hsieh and Klenow (2009). I show that the allocative efficiency term coincides with the allocative efficiency term derived in Hsieh and Klenow (2009). In the closed economy without lobbying, the second term can be rewritten as follows:

$$M^{-\frac{\sigma}{\sigma-1}}\left[\int \frac{1}{\phi}\left(\frac{p(\psi)}{P}\right)^{-\sigma} d\hat{G}(\psi)\right]^{-1}.$$

Using the ideal price index, this can be rewritten as follows:

$$\frac{\left[\int (\phi\tau)^{\sigma-1} d\hat{G}(\psi)\right]^{\frac{1}{\sigma-1}}}{\left[\int \tau \times \underbrace{\frac{(\mu w)^{1-\sigma}(\phi\tau)^{\sigma-1}P^{\sigma-1}E}{E}}_{=\omega(\psi)} d\hat{G}(\psi)\right]},$$

where $\omega(\psi)$ is the share of firms sales' to total expenditures. The denominator is the weighted average of τ where the weights are given by value-added shares of firms. Define $\overline{\text{TFPR}}$ as the denominator of the above expression. Because $\tau \propto$ TFPR, I can obtain the TFP formula of Hsieh and Klenow (2009):

$$A \propto \left[\int \left(\phi \frac{\text{TFPR}}{\overline{\text{TFPR}}}\right)^{\sigma-1} d\hat{G}(\psi)\right]^{\frac{1}{\sigma-1}}.$$

A.3 Proof of Proposition 2.1

This section presents the proof of Proposition 2.1. Without loss of generality, I normalize wage w to one. The price index can be expressed as follows:

$$P^{1-\sigma} = \mu^{\frac{(1-\sigma)(1-\theta)}{1-\theta\sigma}}\left(\frac{\theta}{\kappa}\right)^{\frac{\theta(\sigma-1)}{1-\theta\sigma}}(P^{\sigma}Q)^{\frac{\theta(\sigma-1)}{1-\theta\sigma}}$$
$$\times M_e\left[\tilde{\lambda}(\hat{\phi}^e,\hat{\phi}^x) + (1+\tau_x^{1-\sigma})^{\frac{\theta(\sigma-1)}{1-\theta\sigma}}\tilde{\lambda}(\hat{\phi}^x,\infty) + \tau_x^{1-\sigma}(1+\tau_x^{1-\sigma})^{\frac{\theta(\sigma-1)}{1-\theta\sigma}}\tilde{\lambda}(\hat{\phi}^x,\infty)\right],$$

which can be re-expressed as follows:

$$P^{1-\sigma} = cons \times M_e(P^{\sigma}Q)^{\frac{\theta(\sigma-1)}{1-\theta\sigma}}\frac{\mathcal{L}^{\lambda}}{\lambda}\tilde{\lambda}(\hat{\phi}^e,\infty), \tag{A.7}$$

where $cons$ is a collection of parameters that are invariant to iceberg cost changes, λ is a share of expenditures on domestic varieties (Equation (2.9)) and $\mathcal{L}^{\lambda}$ is the ratio of expenditure on domestic varieties to its counterfactual value when exporters exert lobbying efforts as if they were in autarky (Equation (2.11)). Equation (A.7) is one of the two key equations for the proof.

The free entry condition implies that

$$\begin{aligned} &p_e\Big(\big(\mathbb{J}[\tilde{\pi}^d(b^d)]-(1-p_x)f\big)+\big(\mathbb{J}[\tilde{\pi}^x(b^x)]-p_x f_x-p_x f\big)\Big)=f_e \\ &\Leftrightarrow \mathbb{J}[\tilde{\pi}^d(b^d)]+\mathbb{J}[\tilde{\pi}^x(b^x)]=\Big(\frac{f_e}{p_e}+f+p_x f_x\Big), \end{aligned} \tag{A.8}$$

where $p_e = \frac{1}{1-G(\bar{\phi}^e(\tau,\eta))}$ is the probability of entry, $p_x = \frac{1-G(\bar{\phi}^x(\tau,\eta))}{1-G(\bar{\phi}^e(\tau,\eta))}$ is the probability of exporting conditioning on entry, and $\mathbb{J}[\tilde{\pi}^d(b^d)]$ and $\mathbb{J}[\tilde{\pi}^x(b^x)]$ are defined as follows:

$$\mathbb{J}[\tilde{\pi}^d(b^d)] = \iiint_{\hat{\phi}^e\tau^{\frac{-\sigma}{\sigma-1}}\eta^{\frac{-\theta\sigma}{\sigma-1}}}^{\hat{\phi}^x\tau^{\frac{-\sigma}{\sigma-1}}\eta^{\frac{-\theta\sigma}{\sigma-1}}} \underbrace{(1-\theta\sigma)\left(\frac{\theta\sigma\eta}{w}\right)^{\frac{\theta\sigma}{1-\theta\sigma}}\tilde{\pi}^d(0)^{\frac{1}{1-\theta\sigma}}}_{=\tilde{\pi}^d(b^d)}\, d\hat{G}(\boldsymbol{\psi})$$

$$\mathbb{J}[\tilde{\pi}^x(b^x)] = \iiint_{\hat{\phi}^x\tau^{\frac{-\sigma}{\sigma-1}}\eta^{\frac{-\theta\sigma}{\sigma-1}}}^{\infty} \underbrace{(1-\theta\sigma)\left(\frac{\theta\sigma\eta}{w}\right)^{\frac{\theta\sigma}{1-\theta\sigma}}\tilde{\pi}^x(0)^{\frac{1}{1-\theta\sigma}}}_{=\tilde{\pi}^d(b^d)}\, d\hat{G}(\boldsymbol{\psi}),$$

where $\tilde{\pi}^d(0)$ and $\tilde{\pi}^x(0)$ are operating profits conditional on not lobbying (i.e., operating profits under standard monopolistic competition): $\tilde{\pi}^d(0) = \frac{1}{\sigma}(\frac{\mu}{\phi})^{1-\sigma}\tau^\sigma P^\sigma Q$ and $\tilde{\pi}^x(0) = \frac{1}{\sigma}(\frac{\mu}{\phi})^{1-\sigma}\tau^\sigma(1+\tau_x^{1-\sigma})P^\sigma Q$.

Labor used for production for non-exporters and exporters is

$$\begin{aligned} l_d &= \frac{q_d}{\phi} = (\sigma-1)(\eta\theta\sigma)^{\frac{\theta\sigma}{1-\theta\sigma}}\tilde{\pi}^d(0)^{\frac{1}{1-\theta\sigma}} = \frac{\sigma-1}{1-\theta\sigma}\tilde{\pi}^d(b^d) \\ l^x &= \frac{q_d+\tau_x q^x}{\phi} = (\sigma-1)(\eta\theta\sigma)^{\frac{\theta\sigma}{1-\theta\sigma}}\tilde{\pi}^x(0)^{\frac{1}{1-\theta\sigma}} = \frac{\sigma-1}{1-\theta\sigma}\tilde{\pi}^x(b^x), \end{aligned} \tag{A.9}$$

Labor used for lobbying for non-exporters and exporters is

$$\begin{aligned} \frac{b^d}{\eta} &= \eta^{\frac{\theta\sigma}{1-\theta\sigma}}(\theta\sigma)^{\frac{1}{1-\theta\sigma}}\tilde{\pi}^d(0)^{\frac{1}{1-\theta\sigma}} = \frac{\theta\sigma}{1-\theta\sigma}\tilde{\pi}^d(b^d) \\ \frac{b^x}{\eta} &= \eta^{\frac{\theta\sigma}{1-\theta\sigma}}(\theta\sigma)^{\frac{1}{1-\theta\sigma}}\big(\tilde{\pi}^d(0)+\tilde{\pi}^x(0)\big)^{\frac{1}{1-\theta\sigma}} = \frac{\theta\sigma}{1-\theta\sigma}\tilde{\pi}^x(b^x). \end{aligned} \tag{A.10}$$

Labor market clearing condition implies that

$$M\Big(\mathbb{J}[l^d+\frac{b^d}{\eta}]+\mathbb{J}[l^x+\frac{b^x}{\eta}]+f+p_x f_x\Big)+M_e f_e = L \tag{A.11}$$

Combining Equations (A.9) and (A.10), I can obtain that

$$\left[\frac{\sigma-1}{1-\theta\sigma}+\frac{\theta\sigma}{1-\theta\sigma}\right]\Big(\mathbb{J}[\tilde{\pi}^d(b^d)]+\mathbb{J}[\tilde{\pi}^x(b^x)]\Big) = \mathbb{J}[l^d+\frac{b^d}{\eta}]+\mathbb{J}[l^x+\frac{b^x}{\eta}]. \tag{A.12}$$

Combining the free entry and labor market clearing conditions (Equations (A.8) and (A.12)), firm mass can be expressed as follows:

$$M = \frac{1-\theta\sigma}{\sigma}\frac{L}{(f + p_x f_x + \frac{f_e}{p_e})}. \tag{A.13}$$

Substituting Equations (A.13) and (A.12) into Equation (A.11), I obtain that

$$M\Big(\mathbb{J}[\tilde{\pi}^d(b^d)] + \mathbb{J}[\tilde{\pi}^x(b^x)]\Big) = \frac{\sigma-1+\theta\sigma}{\sigma}L,$$

which can be rewritten as

$$(\sigma-1)(\theta\sigma)^{\frac{\theta\sigma}{1-\theta\sigma}}\left(\frac{\mu}{\sigma}\right)^{\frac{1}{1-\theta\sigma}}\left[\tilde{S}(\hat{\phi}^e,\hat{\phi}^x) + (1+\tau_x^{1-\sigma})^{\frac{\theta\sigma}{1-\theta\sigma}}\tilde{S}(\hat{\phi}^x,\infty) + \tau_x^{1-\sigma}(1+\tau_x^{1-\sigma})^{\frac{\theta\sigma}{1-\theta\sigma}}\tilde{S}(\hat{\phi}^x,\infty)\right]$$
$$\times M_e P^{\frac{\sigma}{1-\theta\sigma}} Q^{\frac{1}{1-\theta\sigma}} = \frac{\sigma-1+\theta\sigma}{\sigma}L.$$

The above expression can be re-expressed as follows:

$$\frac{\mathcal{L}^s}{S}\tilde{S}(\hat{\phi}^e,\infty)M_e P^{\frac{\sigma}{1-\theta\sigma}} Q^{\frac{1}{1-\theta\sigma}} = cons, \tag{A.14}$$

where the right-hand side is a collection of parameters that are invariant to iceberg costs. Equation (A.14) is the second key equation for the proof.

Next, I totally differentiate Equations (A.7) and (A.14). Totally differentiating Equation (A.7) related to the price index, I can obtain the following expression:

$$(1-\sigma)d\ln P = \frac{\sigma\theta(\sigma-1)}{1-\theta\sigma}d\ln P + \frac{\theta(\sigma-1)}{1-\theta\sigma}d\ln Q + d\ln M_e - d\ln\big(\lambda/\mathcal{L}^\lambda\big) - \frac{1}{1-\theta\sigma}\gamma_\lambda(\hat{\phi}^e)d\ln\hat{\phi}^e. \tag{A.15}$$

Similarly, totally differentiating Equation (A.14) related to the labor market clearing and the free entry conditions, I can obtain the following expression:

$$d\ln M_e + \frac{\sigma}{1-\theta\sigma}d\ln P + \frac{1}{1-\theta\sigma}d\ln Q - d\ln(S/\mathcal{L}^s) - \frac{1}{1-\theta\sigma}\gamma_s(\hat{\phi}^e)d\ln\hat{\phi}^e = 0. \tag{A.16}$$

Totally differentiating the entry cutoff,

$$d\ln\hat{\phi}^e = -d\ln P - \frac{1}{\sigma-1}d\ln PQ. \tag{A.17}$$

Combining Equations (A.15) and (A.17), I can derive that

$$-d\ln P = \frac{1}{\gamma_\lambda(\hat{\phi}^e)+(\sigma-1)(1-\theta)}\left\{-d\ln\big(\lambda/\mathcal{L}^\lambda\big) + d\ln M_e\right\} + \frac{\frac{\gamma_\lambda(\hat{\phi}^e)}{\sigma-1}+\theta(\sigma-1)}{\gamma_\lambda(\hat{\phi}^e)+(\sigma-1)(1-\theta)}d\ln PQ. \tag{A.18}$$

Substituting the above equation into $d\ln Q = -d\ln P + d\ln PQ$, because changes in welfare are equivalent to changes in the aggregate quantities produced, $d\ln W = d\ln Q$, I can obtain that

$$d\ln W = \frac{1}{\gamma_\lambda(\hat{\phi}^e) + (\sigma-1)(1-\theta)}\left\{-d\ln\left(\lambda/\mathcal{L}^\lambda\right) + d\ln M_e\right\} + \left(\frac{\frac{\gamma_\lambda(\hat{\phi}^e)}{\sigma-1} + \theta(\sigma-1)}{\gamma_\lambda(\hat{\phi}^e) + (\sigma-1)(1-\theta)} + 1\right) d\ln PQ. \tag{A.19}$$

Combining Equations (A.16) and (A.17),

$$-d\ln P = \frac{1}{\sigma-1} d\ln PQ + \frac{1-\theta\sigma}{\gamma_s(\hat{\phi}^e) + \sigma - 1}\left\{-d\ln(S/\mathcal{L}^s) + d\ln M_e\right\}.$$

Substituting Equation (A.18) into the above equation, I can obtain that

$$d\ln PQ = \frac{1}{1-\theta\sigma}\left\{-d\ln\left(\lambda/\mathcal{L}^\lambda\right) + d\ln M_e\right\} - \frac{\gamma_\lambda + (\sigma-1)(1-\theta)}{\gamma_s + \sigma - 1}\left\{-d\ln(S/\mathcal{L}^s) + d\ln M_e\right\}.$$

Rearranging the equation,

$$\begin{aligned} d\ln PQ = &\left(\frac{1}{1-\theta\sigma} - \frac{\gamma_\lambda(\hat{\phi}^e) + (\sigma-1)(1-\theta)}{\gamma_s(\hat{\phi}^e) + \sigma - 1}\right)\left\{-d\ln\left(\lambda/\mathcal{L}^\lambda\right) + d\ln M_e\right\} \\ &\left(\frac{\gamma_\lambda(\hat{\phi}^e) + (\sigma-1)(1-\theta)}{\gamma_s(\hat{\phi}^e) + \sigma - 1}\right)\left\{-d\ln\left(\lambda/\mathcal{L}^\lambda\right) + d\ln(S/\mathcal{L}^s)\right\}. \end{aligned} \tag{A.20}$$

Combining Equations (A.19) and (A.20) gives the desired result.

□

A.4 Proof of Corollary 2.1

I consider Pareto-distributed productivity with the shape parameter κ with the location parameter normalized to 1. Exogenous distortions and lobbying efficiency are homogeneous. Under the Pareto distribution, I can derive the following set of equations:

$$\begin{aligned} \tilde{\lambda}(\hat{\phi}^e, \infty) &= \int_{\hat{\phi}^e}^{\infty} \phi^{\frac{(\sigma-1)(1-\theta)}{1-\theta\sigma}} \kappa\phi^{-\kappa-1} d\phi = \frac{\kappa}{\kappa - \frac{(\sigma-1)(1-\theta)}{1-\theta\sigma}} (\hat{\phi}^e)^{-\kappa + \frac{(\sigma-1)(1-\theta)}{1-\theta\sigma}} \\ \tilde{S}(\hat{\phi}^e, \infty) &= \int_{\hat{\phi}^e}^{\infty} \phi^{\frac{\sigma-1}{1-\theta\sigma}} \kappa\phi^{-\kappa-1} d\phi = \frac{\kappa}{\kappa - \frac{\sigma-1}{1-\theta\sigma}} (\hat{\phi}^e)^{-\kappa + \frac{\sigma-1}{1-\theta\sigma}} \\ \gamma_\lambda(\hat{\phi}^e) &= -(1-\theta\sigma)\frac{d\ln\tilde{\lambda}(\hat{\phi}^e)}{d\ln\hat{\phi}^e} = (1-\theta\sigma)\kappa - (\sigma-1)(1-\theta) \\ \gamma_s(\hat{\phi}^e) &= -(1-\theta\sigma)\frac{d\ln\tilde{S}(\hat{\phi}^e)}{d\ln\hat{\phi}^e} = (1-\theta\sigma)\kappa - (\sigma-1), \end{aligned}$$

which gives

$$\frac{1}{\gamma_\lambda(\hat{\phi}^e) + (\sigma-1)(1-\theta)} = \frac{1}{(1-\theta\sigma)\kappa}.$$

$$\frac{\gamma_\lambda(\hat{\phi}^e)/(\sigma-1)+\theta(\sigma-1)}{\gamma_\lambda(\hat{\phi}^e)+(\sigma-1)(1-\theta)}+1=\frac{\sigma}{\sigma-1}-\frac{1}{\kappa}.$$

$$\frac{1}{1-\theta\sigma}-\frac{\gamma_\lambda(\hat{\phi}^e)+(\sigma-1)(1-\theta)}{\gamma_s(\hat{\phi}^e)+\sigma-1}=\frac{\theta\sigma}{1-\theta\sigma}.$$

$$\frac{\gamma_\lambda(\hat{\phi}^e)+(\sigma-1)(1-\theta)}{\gamma_s(\hat{\phi}^e)+\sigma-1}=1.$$

Now, I show that $d\ln M_e=0$. The free entry condition (Equation (A.8)) can be written as follows:

$$\int_{\hat{\phi}^e}^{\hat{\phi}^x}\left(\frac{\phi}{\hat{\phi}^e}\right)^{\frac{\sigma-1}{1-\theta\sigma}}\tilde{\pi}^d(\hat{\phi}^e)dG(\phi)+\int_{\hat{\phi}^x}^{\infty}\left(\frac{\phi}{\hat{\phi}^x}\right)^{\frac{\sigma-1}{1-\theta\sigma}}\tilde{\pi}^x(\hat{\phi}^x)dG(\phi)-p_e f-p_e p_x f_x=f_e,$$

where I used that operating profits of non-exporters and exporters can be written in terms of the other firm: $\tilde{\pi}^o(\phi)=(\phi/\phi')^{\frac{\sigma-1}{1-\theta\sigma}}\tilde{\pi}^o(\phi')$ for $o\in\{d,x\}$. Using that $\tilde{\pi}^d(\hat{\phi}^e)=f$ and $\tilde{\pi}^x(\hat{\phi}^x)=\tilde{\pi}^d(\hat{\phi}^x)+f_x=(\hat{\phi}^x/\hat{\phi}^e)^{\frac{\sigma-1}{1-\theta\sigma}}f+f_x$,

$$f\int_{\hat{\phi}^e}^{\infty}\left(\frac{\phi}{\hat{\phi}^e}\right)^{\frac{\sigma-1}{1-\theta\sigma}}dG(\phi)+f_x\int_{\hat{\phi}^e}^{\hat{\phi}^x}\left(\frac{\phi}{\hat{\phi}^e}\right)^{\frac{\sigma-1}{1-\theta\sigma}}dG(\phi)-p_e f-p_e p_x f_x=f_e\Leftrightarrow p_e f+p_e p_x f_x=\frac{\kappa-\frac{\sigma-1}{1-\theta\sigma}}{\frac{\sigma-1}{1-\theta\sigma}}f_e.$$

Substituting the above expression into Equation (A.13) and using that $p_e M_e=M$, I obtain that $M_e=\frac{\sigma-1}{\sigma\kappa}\frac{L}{f_e}$, which is a function of only parameters and therefore remains constant regardless of values of iceberg costs. Combining these results, the welfare formula becomes

$$\begin{aligned}d\ln\mathbb{W}=&\frac{1}{\kappa}\big\{-d\ln\lambda\big\}+\frac{1}{\kappa}\big\{d\ln\mathcal{L}^\lambda\big\}\\&+\left(\frac{\sigma}{\sigma-1}\frac{1}{1-\theta\sigma}-\frac{1}{\kappa}\right)\big\{-d\ln\lambda+d\ln\mathcal{L}^\lambda\big\}+\left(\frac{\sigma}{\sigma-1}-\frac{1}{\kappa}\right)\big\{d\ln S-d\ln\mathcal{L}^s\big\}.\end{aligned}$$

It remains to show signs of each term in the above equation. It is obvious that the first two terms are always positive when moving from autarky to an open economy. Therefore, it suffices to show that the last term is negative when fixed export costs are sufficiently high. Note that the following holds:

$$\begin{aligned}&\left(\frac{\sigma}{\sigma-1}\frac{1}{1-\theta\sigma}-\frac{1}{\kappa}\right)\big\{-d\ln\lambda+d\ln\mathcal{L}^\lambda\big\}+\left(\frac{\sigma}{\sigma-1}-\frac{1}{\kappa}\right)\big\{d\ln S-d\ln\mathcal{L}^s\big\}<0\\&\Leftrightarrow(\lambda/\mathcal{L}^\lambda)^{\frac{\sigma}{\sigma-1}\frac{1}{1-\theta\sigma}-\frac{1}{\kappa}}>(S/\mathcal{L}^s)^{\frac{\sigma}{\sigma-1}-\frac{1}{\kappa}}.\end{aligned}$$

Note that $\frac{\sigma}{\sigma-1}\frac{1}{1-\theta\sigma}-\frac{1}{\kappa}>0$ and $\frac{\sigma}{\sigma-1}-\frac{1}{\kappa}>0$ holds under the regularity conditions. Also, note that

$$\frac{\lambda}{\mathcal{L}^\lambda}=\frac{\tilde{\lambda}(\hat{\phi}^e,\infty)}{\tilde{\lambda}(\hat{\phi}^e,\infty)+[(1+\tau_x^{1-\sigma})^{\frac{1-\theta}{1-\theta\sigma}}-1]\tilde{\lambda}(\hat{\phi}^x,\infty)}=\frac{1}{1+[(1+\tau_x^{1-\sigma})^{\frac{1-\theta}{1-\theta\sigma}}-1](\frac{\hat{\phi}^x}{\hat{\phi}^e})^{\frac{\sigma-1}{\sigma}(-\kappa+\frac{\sigma-1}{1-\theta\sigma})}}.$$

$$\frac{S}{\mathcal{L}^s} = \frac{\tilde{S}(\hat{\phi}^e, \infty)}{\tilde{S}(\hat{\phi}^e, \infty) + [(1+\tau_x^{1-\sigma})^{\frac{1}{1-\theta\sigma}} - 1]\tilde{S}(\hat{\phi}^x, \infty)} = \frac{1}{1 + [(1+\tau_x^{1-\sigma})^{\frac{1}{1-\theta\sigma}} - 1](\frac{\hat{\phi}^x}{\hat{\phi}^e})^{\frac{\sigma-1}{\sigma}(-\kappa + \frac{\sigma}{1-\theta\sigma})}}.$$

In autarky, $\lambda/\mathcal{L}^\lambda = S/\mathcal{L}^s = 1$. However, because $\frac{\hat{\phi}^x}{\hat{\phi}^e} = \left(\frac{f_x}{f}\right)^{\frac{1-\theta\sigma}{\sigma-1}} \frac{1}{[(1+\tau_x^{1-\sigma})^{\frac{1}{1-\theta\sigma}}-1]^{\frac{1-\theta\sigma}{\sigma-1}}} > 1$ holds (i.e., the condition under which selection into exporting occurs), $1 > \lambda/\mathcal{L}^\lambda > S/\mathcal{L}^s$ for $\forall \tau_x \in [1, \infty)$. As fixed export cost f_x increases, the gap between the two objects $\lambda/\mathcal{L}^\lambda - S/\mathcal{L}^s$ becomes higher for given values of τ_x. Therefore, there exists a sufficiently high f_x that makes $(\lambda/\mathcal{L}^\lambda)^{\frac{\sigma}{\sigma-1}\frac{1}{1-\theta\sigma}-\frac{1}{\kappa}} > (S/\mathcal{L}^s)^{\frac{\sigma}{\sigma-1}-\frac{1}{\kappa}}$ holds for a given τ_x, which is equivalent to the condition under which the last term becomes negative.

□

A.5 Two Special Cases

In this subsection, I consider two additional special cases: (a) τ is Pareto-distributed, and ϕ and η are homogeneous across firms; and (b) η is Pareto-distributed, and ϕ and τ are homogeneous. Corollary A.1 presents the welfare formula under these two cases

Corollary A.1. *Consider two special case: (a) τ follows a Pareto distribution with the shape parameter κ, with ϕ and η homogeneous across firms; and (b) η follows a Pareto distribution with the shape parameter κ, with ϕ and τ homogeneous across firms. Suppose Assumption 2.1 holds. When moving from autarky to an open economy, the welfare effects of trade are given by*

$$d \ln \mathbb{W} = \left(\frac{\sigma}{\sigma-1}\frac{1}{1-\theta\sigma}\right)\left\{-d\ln\lambda + d\ln\mathcal{L}^\lambda\right\} + \left(\frac{\sigma}{\sigma-1}\right)\left\{d\ln S - d\ln\mathcal{L}^s\right\}. \tag{A.21}$$

A.5.1 Proof of Corollary 2.1

Pareto exogenous distortion. The entry and export cutoff distortions are given by $(\hat{\tau}^e)^{\frac{\sigma}{\sigma-1}} = \hat{\phi}^e$ and $(\hat{\tau}^x)^{\frac{\sigma}{\sigma-1}} = \hat{\phi}^x$. Under the Pareto distribution, I can derive the following set of equations:

$$\tilde{\lambda}(\hat{\phi}^e) = \int_{(\hat{\phi}^e)^{\frac{\sigma-1}{\sigma}}}^{\infty} \tau^{-\kappa+\frac{\sigma-1}{1-\theta\sigma}-1}\kappa d\tau = \frac{\kappa}{\kappa - \frac{\sigma-1}{1-\theta\sigma}}(\hat{\phi}^e)^{\frac{\sigma-1}{\sigma}(-\kappa+\frac{\sigma-1}{1-\theta\sigma})}$$

$$\tilde{S}(\hat{\phi}^e, \infty) = \int_{(\hat{\phi}^e)^{\frac{\sigma-1}{\sigma}}} \tau^{-\kappa+\frac{\sigma}{1-\theta\sigma}-1}\kappa d\tau = \frac{\kappa}{\kappa - \frac{\sigma}{1-\theta\sigma}}(\hat{\phi}^e)^{\frac{\sigma-1}{\sigma}(-\kappa+\frac{\sigma}{1-\theta\sigma})}.$$

$$\gamma_\lambda(\hat{\phi}^e) = \frac{\sigma-1}{\sigma}((1-\theta\sigma)\kappa - (\sigma-1))$$

$$\gamma_s(\hat{\phi}^e) = \frac{\sigma-1}{\sigma}((1-\theta\sigma)\kappa - \sigma),$$

$$\frac{1}{\gamma_\lambda(\hat{\phi}^e) + (\sigma-1)(1-\theta)} = \frac{\sigma}{\sigma-1}\frac{1}{1-\theta\sigma}\frac{1}{\kappa+1}$$

$$\frac{\gamma_\lambda(\hat{\phi}^e)/(\sigma-1) + \theta(\sigma-1)}{\gamma_\lambda(\hat{\phi}^e) + (\sigma-1)(1-\theta)} + 1 = \frac{\sigma}{\sigma-1}\frac{\kappa}{\kappa+1}.$$

$$\frac{1}{1-\theta\sigma}-\frac{\gamma_\lambda(\hat{\phi}^e)+(\sigma-1)(1-\theta)}{\gamma_s(\hat{\phi}^e)+\sigma-1}=\frac{1}{1-\theta\sigma}-\frac{\kappa+1}{\kappa}$$

$$\frac{\gamma_\lambda(\hat{\phi}^e)+(\sigma-1)(1-\theta)}{\gamma_s(\hat{\phi}^e)+\sigma-1}=\frac{\kappa+1}{\kappa}.$$

Similar to the derivation in the case of Pareto-productivity, using that

$$f\int_{\hat{\tau}^e}^{\infty}(\tau/\hat{\tau}^e)^{\frac{\sigma}{1-\theta\sigma}}dG(\tau)+f_x\int_{\hat{\tau}^x}^{\infty}(\tau/\hat{\tau}^x)^{\frac{\sigma}{1-\theta\sigma}}dG(\tau)-p_e f-p_e p_x f_x=f_e,$$

I can obtain that $p_e f+p_e p_x f_x=f_e\frac{\kappa-\sigma/(1-\theta\sigma)}{\sigma/(1-\theta\sigma)}$ and substituting this into Equation (A.13) and using that $p_e M_e=M$, the entry mass can be expressed as $M_e=\frac{L}{\kappa f_e}$, which is a function of only parameters and therefore $d\ln M_e=0$. Combining these results, the welfare formula becomes

$$d\ln\mathbb{W}=\frac{\sigma}{\sigma-1}\frac{1}{1-\theta\sigma}\{-d\ln\lambda+d\ln\mathcal{L}^\lambda\}+\frac{\sigma}{\sigma-1}\{d\ln S-d\ln\mathcal{L}^s\}.$$

Pareto lobbying efficiency. The entry and export lobbying efficiency cutoffs are given by $\hat{\eta}^e=(\hat{\phi}^e)^{\frac{\sigma-1}{\theta\sigma}}$ and $\hat{\eta}^x=(\hat{\phi}^x)^{\frac{\sigma-1}{\theta\sigma}}$. Under the Pareto-distribution, I obtain the following set of equations:

$$\tilde{\lambda}(\hat{\phi}^e,\infty)=\int_{(\hat{\phi}^e)^{\frac{\sigma-1}{\theta\sigma}}}^{\infty}\kappa\eta^{-\kappa+\frac{\theta(\sigma-1)}{1-\theta\sigma}-1}d\eta=\frac{\kappa}{\kappa-\frac{\theta(\sigma-1)}{1-\theta\sigma}}(\hat{\phi}^e)^{\frac{\sigma-1}{\theta\sigma}(-\kappa+\frac{\theta(\sigma-1)}{1-\theta\sigma})}.$$

$$\tilde{S}(\hat{\phi}^e,\infty)=\int_{(\hat{\phi}^e)^{\frac{\sigma-1}{\theta\sigma}}}^{\infty}\kappa\eta^{-\kappa+\frac{\theta\sigma}{1-\theta\sigma}-1}d\eta=\frac{\kappa}{\kappa-\frac{\theta\sigma}{1-\theta\sigma}}(\hat{\phi}^e)^{\frac{\sigma-1}{\theta\sigma}(-\kappa+\frac{\theta\sigma}{1-\theta\sigma})}.$$

$$\gamma_\lambda(\hat{\phi}^e)=\frac{\sigma-1}{\theta\sigma}((1-\theta\sigma)\kappa-\theta(\sigma-1))\qquad\text{and}\qquad\gamma_s(\hat{\phi}^e)=\frac{\sigma-1}{\theta\sigma}((1-\theta\sigma)\kappa-\theta\sigma).$$

$$\frac{1}{\gamma_\lambda(\hat{\phi}^e)+(\sigma-1)(1-\theta)}=\frac{\sigma}{\sigma-1}\frac{\theta}{1-\theta\sigma}\frac{1}{\kappa+\theta}.$$

$$\frac{\gamma_\lambda(\hat{\phi}^e)/(\sigma-1)+\theta(\sigma-1)}{\gamma_\lambda(\hat{\phi}^e)+(\sigma-1)(1-\theta)}+1=\frac{\sigma}{\sigma-1}\frac{\kappa}{\kappa+\theta}.$$

$$\frac{1}{1-\theta\sigma}-\frac{\gamma_\lambda(\hat{\phi}^e)+(\sigma-1)(1-\theta)}{\gamma_s(\hat{\phi}^e)+\sigma-1}=\frac{1}{1-\theta\sigma}-\frac{\kappa+\theta}{\kappa}.$$

$$\frac{\gamma_\lambda(\hat{\phi}^e)+(\sigma-1)(1-\theta)}{\gamma_s(\hat{\phi}^e)+\sigma-1}=\frac{\kappa+\theta}{\kappa}.$$

To show that $d\ln M_e=0$, similar to the previous case, using the modified free entry condition,

$$f\int_{\hat{\eta}^e}^{\infty}(\eta/\hat{\eta}^e)^{\frac{\theta\sigma}{1-\theta\sigma}}dG(\eta)+f_x\int_{\hat{\eta}^x}^{\infty}(\eta/\hat{\eta}^x)^{\frac{\theta\sigma}{1-\theta\sigma}}dG(\eta)-p_e f-p_e p_x f_x=f_e,$$

I can obtain that $p_e f+p_e p_x f_x=\frac{\kappa-\theta\sigma/(1-\theta\sigma)}{\theta\sigma/(1-\theta\sigma)}$. Substituting this expression into Equation (A.13), I obtain that $M_e=\frac{\theta L}{\kappa f_e}$, which is a function of only parameters and therefore $d\ln M_e=0$. Combining these results, the welfare formula is expressed as follows:

$$d\ln\mathbb{W}=\frac{\sigma}{\sigma-1}\frac{1}{1-\theta\sigma}\{-d\ln\lambda+d\ln\mathcal{L}^{\lambda}\}+\frac{\sigma}{\sigma-1}\{d\ln S-d\ln\mathcal{L}^{s}\}.$$

□

B. Quantification Appendix

B.1 Estimation of the Elasticity of Output Distortions to Lobbying

B.1.1 Direction of the OLS Bias

The direction of bias of the OLS estimates $\hat{\beta}^{\text{OLS}}$ in Equation (3.1) can be interpreted through the lens of the model. The bias is affected by covariances and variances of firm primitives. For exposition purposes, I will consider the regression model without any controls and a simplified closed economy setup in which every firm is operating and lobbying, which can be achieved by setting $f_b=0$, $f=0$, and $\tau_x\to\infty$. Under this setup, selection into production, exporting, and lobbying do not affect the bias and the bias can be expressed as follows:

$$\hat{\beta}^{\text{OLS}}\xrightarrow{p}\theta+\underbrace{\frac{\text{Cov}(\ln Lobby_{it},\ln\tau_{it}+\theta\ln\eta_{it})}{\text{Var}(\ln Lobby_{it})}}_{=\mathcal{B}(\ln\psi_{it})},$$

where $\mathcal{B}(\ln\psi_{it})$ is the bias that is a function of covariances and variances of firm primitives:

$$\begin{aligned}\mathcal{B}(\ln\psi_{it})=&\frac{1}{\text{Var}(\ln Lobby_{it})}\Big(\frac{\theta^2\sigma}{1-\theta\sigma}\text{Var}(\ln\eta_{it})+\frac{\sigma}{1-\theta\sigma}\text{Var}(\ln\tau_{it})\\&+\frac{2\theta\sigma}{1-\theta\sigma}\text{Cov}(\ln\tau_{it},\ln\eta_{it})+\frac{\sigma-1}{1-\theta\sigma}\text{Cov}(\ln\phi_{it},\ln\tau_{it})+\frac{\theta(\sigma-1)}{1-\theta\sigma}\text{Cov}(\ln\phi_{it},\ln\eta_{it})\Big),\end{aligned}\quad\text{(B.1)}$$

where

$$\begin{aligned}\text{Var}(\ln Lobby_{it})=&\left(\frac{\theta\sigma}{1-\theta\sigma}\right)^2\text{Var}(\ln\eta_{it})+\left(\frac{\sigma}{1-\theta\sigma}\right)^2\text{Var}(\ln\tau_{it})+\left(\frac{\sigma-1}{1-\theta\sigma}\right)^2\text{Var}(\ln\phi_{it})\\&+\frac{2\sigma(\sigma-1)}{(1-\theta\sigma)^2}\text{Cov}(\ln\phi_{it},\ln\tau_{it})+\frac{2\theta\sigma(\sigma-1)}{(1-\theta\sigma)^2}\text{Cov}(\ln\phi_{it},\ln\eta_{it})+\frac{2\theta\sigma^2}{(1-\theta\sigma)^2}\text{Cov}(\ln\eta_{it},\ln\tau_{it}).\end{aligned}$$

Depending on signs of the covariances, the bias can take both positive and negative values. If the covariances are sufficiently negative, the OLS estimate will be downward biased, as in Table 2. Based on the calibrated values and the estimates of the variances and covariances reported in Table 3 in the later section, the bias is −0.04, consistent with the downward bias.

Similarly, the bias of the sales regression model in Equation (3.2) can be expressed as follows:

$$\mathcal{B}^s(\psi_{it}) = \frac{1}{\text{Var}(\ln Lobby_{it})} \times \text{Cov}\left(\frac{\sigma-1}{1-\theta\sigma}\ln\phi_{it} + \frac{\sigma}{1-\theta\sigma}\ln\tau_{it} + \frac{\theta\sigma}{1-\theta\sigma}\ln\eta_{it}, (\sigma-1)\ln\phi_{it} + \sigma\ln\tau_{it} + \theta\sigma\ln\eta_{it}\right), \quad \text{(B.2)}$$

where

$$\begin{aligned}\text{Cov}&\left(\frac{\sigma-1}{1-\theta\sigma}\ln\phi_{it} + \frac{\sigma}{1-\theta\sigma}\ln\tau_{it} + \frac{\theta\sigma}{1-\theta\sigma}\ln\eta_{it}, (\sigma-1)\ln\phi_{it} + \sigma\ln\tau_{it} + \theta\sigma\ln\eta_{it}\right)\\ &= \frac{(\sigma-1)^2}{1-\theta\sigma}\text{Var}(\ln\phi_{it}) + \frac{\sigma^2}{1-\theta\sigma}\text{Var}(\ln\tau_{it}) + \frac{(\theta\sigma)^2}{1-\theta\sigma}\text{Var}(\ln\eta_{it})\\ &+ \frac{2\sigma(\sigma-1)}{1-\theta\sigma}\text{Cov}(\ln\phi_{it}, \ln\tau_{it}) + \frac{2\theta\sigma(\sigma-1)}{1-\theta\sigma}\text{Cov}(\ln\phi_{it}, \ln\eta_{it}) + \frac{2\theta\sigma^2}{1-\theta\sigma}\text{Cov}(\ln\tau_{it}, \ln\eta_{it}).\end{aligned}$$

Derivation of Equation (B.1). I derive the expression in Equation (B.1). In the simplified setup, firms' optimal lobbying inputs are expressed as

$$b_{it} \propto \eta_{it}^{\frac{1}{1-\theta\sigma}}\phi_{it}^{\frac{\sigma-1}{1-\theta\sigma}}\tau_{it}^{\frac{\sigma}{1-\theta\sigma}}.$$

Because $Lobby_{it} = b_{it}/\eta_{it}$,

$$Lobby_{it} \propto \eta_{it}^{\frac{\theta\sigma}{1-\theta\sigma}}\phi_{it}^{\frac{\sigma-1}{1-\theta\sigma}}\tau_{it}^{\frac{\sigma}{1-\theta\sigma}}.$$

Using the above equation, $\text{Cov}(\ln Lobby_{it}, \ln\tau_{it} + \theta\ln\eta_{it})$ can be expressed as

$$\text{Cov}(\ln Lobby_{it}, \ln\tau_{it} + \theta\ln\eta_{it}) = \text{Cov}(\frac{\theta\sigma}{1-\theta\sigma}\ln\eta_{it} + \frac{\sigma-1}{1-\theta\sigma}\ln\phi_{it} + \frac{\sigma}{1-\theta\sigma}\ln\tau_{it}, \ln\tau_{it} + \theta\ln\eta_{it})$$

which can be rearranged to

$$\begin{aligned}\text{Cov}(\ln Lobby_{it}, \ln\tau_{it} + \theta\ln\eta_{it}) &= \frac{\theta^2\sigma}{1-\theta\sigma}\text{Var}(\ln\eta_{it}) + \frac{\sigma}{1-\theta\sigma}\text{Var}(\ln\tau_{it})\\ &+ \frac{2\theta\sigma}{1-\theta\sigma}\text{Cov}(\ln\tau_{it}, \ln\eta_{it}) + \frac{\sigma-1}{1-\theta\sigma}\text{Cov}(\ln\phi_{it}, \ln\tau_{it}) + \frac{\theta(\sigma-1)}{1-\theta\sigma}\text{Cov}(\ln\phi_{it}, \ln\eta_{it}).\end{aligned}$$

$\text{Var}(\ln Lobby_{it})$ can be expressed as

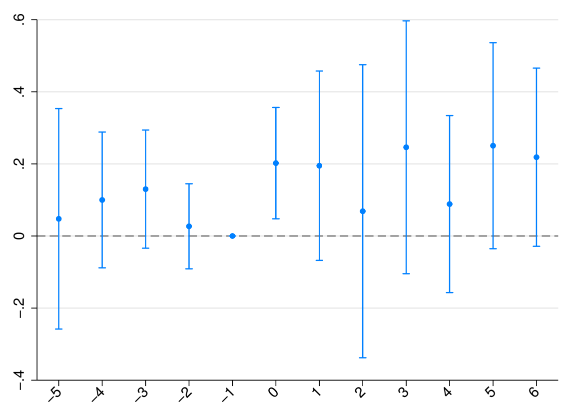
$$\begin{aligned}\text{Var}(\ln Lobby_{it}) &= \left(\frac{\theta\sigma}{1-\theta\sigma}\right)^2\text{Var}(\ln\eta_{it}) + \left(\frac{\sigma}{1-\theta\sigma}\right)^2\text{Var}(\ln\tau_{it}) + \left(\frac{\sigma-1}{1-\theta\sigma}\right)^2\text{Var}(\ln\phi_{it})\\ &+ \frac{2\sigma(\sigma-1)}{(1-\theta\sigma)^2}\text{Cov}(\ln\phi_{it}, \ln\tau_{it}) + \frac{2\theta\sigma(\sigma-1)}{(1-\theta\sigma)^2}\text{Cov}(\ln\phi_{it}, \ln\eta_{it}) + \frac{2\theta\sigma^2}{(1-\theta\sigma)^2}\text{Cov}(\ln\eta_{it}, \ln\tau_{it})\end{aligned}$$

B.1.2 Event Study

One concern is that the first-stage results may reflect spurious correlations between lobbying expenditures and firm primitives rather than causality. Although the exclusion restriction is untestable, an event study can detect spurious correlations caused by reverse causality problems or preexisting confounding factors by checking pre-trends. For example, a reverse causality problem can arise if a firm lobbies to make a local Congress member be appointed as a chairperson in the Appropriations Committee. I estimate the following event study specification:

$$\ln Lobby_{it} = \sum_{\tau=-5}^{5} \beta_\tau \text{Chair}_{i\tau} + \delta_i + \delta_{jt} + \varepsilon_{it}. \tag{B.3}$$

The dependent variable is log lobbying expenditures, with zero values assigned for observations with zero expenditures ($\ln Lobby_{it}$). $\text{Chair}_{i,t-\tau}$ are the event study variables defined as $\text{Chair}_{i,\tau} = \mathbb{1}[t = t_i^{\text{Chair}} - \tau]$, where t_i^{Chair} is the year when a local Congress member of the state in which firm i is headquartered is appointed as the chairperson. $\text{Chair}_{i,-1}$ is normalized to be zero, so β_τ is interpreted as the changes of lobbying expenditures relative to the one year before the appointment. The samples include both treated and non-treated firms. Firm fixed effects δ_i and sector-time fixed effects δ_{jt} are controlled to absorb time-invariant unobservables and sectoral shocks. Standard errors are clustered at the state-level.



Dep. $\ln Lobby_{it}$

Figure B.1: Event Study. Lobbying and Appointment as Chairperson of the House or Senate Appropriations Committee

Notes. This figure illustrates event study coefficients β_τ in Equation (B.3). The dependent variables are log lobbying expenditures. The coefficient in $t-1$ is normalized to be zero. The specification includes firm fixed effects, sector-year fixed effects, and the initial lobbying status interacted with year fixed effects. Standard errors are clustered at the state level. The vertical lines show the 90% confidence intervals.

Figure B.1 illustrates estimated coefficients β_τ in Equation (B.3). Before the events, there are no pre-trends in lobbying expenditures, but once a local Congress member becomes the chairperson, firms start increasing their lobbying expenditures. The evidence of no pre-trends supports the identifying assumption.

B.2 Identifying Moments

This section describes how the identifying moment in the data can be mapped to the data counterparts. In the calibration procedure, the internally calibrated parameters are all jointly determined, but I describe the identifying moment that is most relevant for each parameter.

- Mean productivity of the US relative to that of Foreign, $\mu_\phi^{\text{US}}/\mu_\phi^F$
  - I normalize the mean productivity of Foreign to be one $\mu_\phi^F = 1$. I define the real GDP as:

$$\text{Real GDP} = \frac{M\Big(\int r(\psi)d\hat{G}(\psi) + \int x(\psi)r^x(\psi)d\hat{G}(\psi)\Big)}{M\Big(\int p(\psi)^{1-\sigma}d\hat{G}(\psi)\Big)^{\frac{1}{1-\sigma}}},$$

    where r and r^x are domestic and export revenues, and the denominator is the defined PPI. Holding other parameters constant, the mean productivity of the US increases the US real GDP; therefore, this moment can pin down μ_ϕ^{US}.

- Standard deviation of log productivity, σ_ϕ
  - ϕ can be mapped to TFPQ in the data:

$$\phi \propto \text{TFPQ} = \frac{(\text{Value-Added})^{\frac{\sigma}{\sigma-1}}}{wL}.$$

    Therefore, the variance of the log TFPQ can pin down σ_ϕ.

- Standard deviation of log exogenous distortions, σ_τ
  - The residuals from Equation (3.1) can be mapped to $\theta \ln \eta + \ln \tau$. Therefore, the variance of this residual can be mapped to

$$\theta^2\sigma_\eta^2 + \theta\rho_{\tau\eta}\sigma_\eta\sigma_\tau + \sigma_\tau^2.$$

    The above relationship shows that conditional on θ, σ_η, and $\rho_{\tau\eta}$, the variance of the residuals is informative on σ_τ.

- Standard deviation of log lobbying efficiency, σ_η
  - The log lobbying expenditures in dollar terms ($B_{it} = \frac{\kappa w b}{\eta}$) is proportional to

$$B_{it} \propto \frac{1}{1-\theta\sigma}((\sigma-1)\ln\phi + \sigma\ln\tau + \theta\sigma\ln\eta).$$

    Therefore, the variance of log lobbying expenditures can be mapped to

$$\frac{1}{(1-\theta\sigma)^2}\Big((\sigma-1)^2\sigma_\phi^2 + \sigma^2\sigma_\tau^2 + (\theta\sigma)^2\sigma_\eta^2 \\ + 2(\sigma-1)\sigma\rho_{\phi\tau}\sigma_\phi\sigma_\tau + 2(\sigma-1)\theta\sigma\rho_{\phi\eta}\sigma_\phi\sigma_\eta + 2\sigma(\theta\sigma)\rho_{\tau\eta}\sigma_\tau\sigma_\eta\Big),$$

    which is informative on σ_η conditioning on the other parameters.

- Correlation between log productivity and log exogenous distortions, $\rho_{\phi\tau}$
  - The correlation between the log of TFPQ and the residuals from Equation (3.1) can be mapped to $\theta\rho_{\phi\eta} + \rho_{\phi\tau}$, which is informative on $\rho_{\phi\tau}$.
- Correlation between log productivity and log lobbying efficiency, $\rho_{\phi\eta}$
  - The correlation between TFPQ and firm lobbying expenditures in dollar terms ($B_{it} = \frac{\kappa w b}{\eta}$) can be mapped to
$$\frac{\sigma-1}{1-\theta\sigma}\sigma_\phi^2 + \frac{\sigma}{1-\theta\sigma}\rho_{\phi\tau} + \frac{\theta\sigma}{1-\theta\sigma}\rho_{\phi\eta}.$$
- Correlation between log exogenous distortions and log lobbying efficiency, $\rho_{\tau\eta}$
  - The correlation between the residuals from Equation (3.1) and lobbying expenditures can be mapped to the numerator of the bias expressed in Equation (B.1):
$$\frac{\theta^2\sigma}{1-\theta\sigma}\sigma_\eta^2 + \frac{\sigma}{1-\theta\sigma}\sigma_\tau^2 + \frac{\theta(\sigma-1)}{1-\theta\sigma}\sigma_\phi\sigma_\eta\rho_{\phi\eta} + \frac{2\theta\sigma}{1-\theta\sigma}\sigma_\tau\sigma_\eta\rho_{\tau\eta} + \frac{\sigma-1}{1-\theta\sigma}\sigma_\phi\sigma_\tau\rho_{\phi\tau}.$$
- Parameter related to the level of variable lobbying cost, κ
  - To identify this parameter, I target the fraction of the median sales of lobbying firms to those of non-lobbying firms:
$$\frac{\text{Median}_{\{\psi|\phi\geq\bar{\phi}^b(\tau,\eta)\}}\{r(b;\psi)\}}{\text{Median}_{\{\psi|\phi<\bar{\phi}^b(\tau,\eta)\}}\{r(0;\psi)\}},$$
    where $r(b;\psi)$ and $r(0;\psi)$ are lobbying and non-lobbying firms' sales, respectively. Because κ only appears in lobbying firms' sales, this moment can pin down κ.
- Fixed lobbying costs, f_b
  - f_b affects extensive margin of lobbying (Equations (A.3) and (A.4)). By targeting the probability of participating in lobbying, I can pin down f_b.
- Fixed export costs, f_x
  - f_x affects extensive margin of exporting (Equations (A.5) and (A.6)). By targeting the probability of participating in exporting, I can pin down f_x.
- Fixed production costs, f
  - f affects production decisions of firms. Because only small-sized firms are affected by f, the difference between the median and 10p of log sales can pin down this parameter.
- Iceberg costs, τ_x
  - The aggregate US import shares can be expressed as follows:
$$\frac{M_f\left[\int x_f(\psi)\left(\frac{\mu\tau_x w_f}{\phi}\right)^{1-\sigma}\tau^\sigma \hat{G}_f(\psi)\right]P^{\sigma-1}E}{E}.$$
    Holding other variables constant, higher τ_x decreases the US import shares. Therefore, the US import shares pin down τ_x.

B.3 Algorithm

I describe an algorithm used for the method of moments.

Step 1. Guess a set of parameters.

Step 2. Based on the guess, simulate 250,000 number of firms whose primitives are randomly drawn from joint distributions based on the initial guess.

Step 3. Based on the 250,000 draws, solve for the equilibrium:

- The wage of Home is normalized to 100
- Guess five aggregate variables: $\{P^{(0)}, E^{(0)}, w_f^{(0)}, P_f^{(0)}, E_f^{(0)}\}$.
- Given this guess for the five unknowns, compute individual firms' optimal entry, production, exporting, and lobbying decisions.
- Using individual firms' decisions, compute firm mass using labor market clearing condition:

$$M = \frac{L}{\int \Big(l(\psi) + f + x(\psi) f_x + \kappa \frac{b(\psi)}{\eta} + \mathbb{1}[b(\psi) > 0] f_b\Big) d\hat{G}(\psi) + \frac{f_e}{p_e}}$$

  and transfers

$$T = M\Bigg[\int \big(1 - \tau^y(\psi)\big)\Big(p(\psi)q(\psi) + x(\psi)p^x(\psi)q^x(\psi)\Big) d\hat{G}(\psi)\Bigg].$$

  T_f and M_f can be obtained similarly using equilibrium conditions for Foreign.

- Check whether individual firms' optimal decisions are consistent with the guessed five aggregate variables: the price indices for both Home and Foreign

$$(P^{(0)})^{1-\sigma} = M\Bigg[\int p(\psi)^{1-\sigma} d\hat{G}(\psi)\Bigg] + M_f\Bigg[\int x(\psi)p(\psi)^{1-\sigma} d\hat{G}_f(\psi)\Bigg] \tag{B.4}$$

$$(P_f^{(0)})^{1-\sigma} = M_f\Bigg[\int p(\psi)^{1-\sigma} d\hat{G}_f(\psi)\Bigg] + M\Bigg[\int x(\psi)p(\psi)^{1-\sigma} d\hat{G}(\psi)\Bigg], \tag{B.5}$$

  the goods market clearing conditions for both Home and Foreign

$$E^{(0)} = wL + T \qquad \text{and} \qquad E_f^{(0)} = w_f^{(0)} L_f + T_f \tag{B.6}$$

  and the balanced trade condition

$$M\Bigg[\int x(\psi)p^x(\psi)q^x(\psi) d\hat{G}(\psi)\Bigg] = M_f\Bigg[\int x_f(\psi)p_f^x(\psi)q_f^x(\psi) d\hat{G}_f(\psi)\Bigg]. \tag{B.7}$$

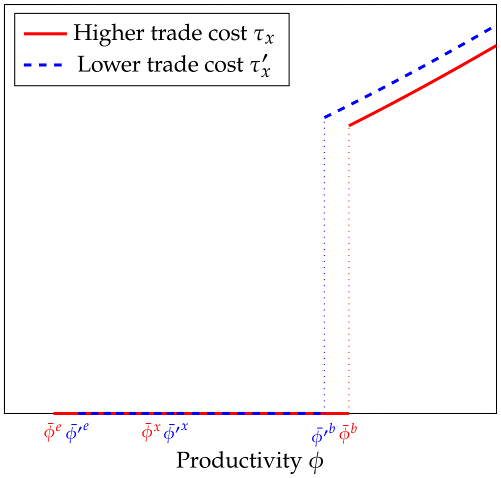
- Using the nonlinear solver, find $\{P^{(0)}, E^{(0)}, w_f^{(0)}, P_f^{(0)}, E_f^{(0)}\}$ that satisfies the above five nonlinear equations (Equations (B.4), (B.5), (B.6), (B.7)).

Step 4. Evaluate the moments computed from the model and compare these moments to the data counterparts.

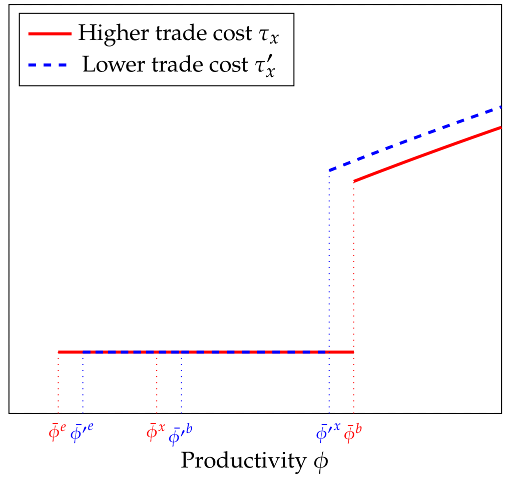
Step 5. I first look for a range of plausible values of parameters using grid search. I repeat steps 1-4 for a given grid.

Step 6. Once I find a range of plausible values of parameters, I find the parameter that minimizes the objective function subject to this range using the constrained nonlinear optimization algorithm where the constraint is given by step 3.

B.4 Additional Figures and Tables



A. Lobbying expenditure, $w\kappa\frac{b}{\eta}$



B. Output distortion, τ^y

Figure B.2: Lower trade costs lead exporters to increase their lobbying efforts.

Notes. This figure illustrates changes in firm lobbying and output distortions depending on their productivity level and changes in the entry, export, and lobbying cutoffs when trade costs become lower. This figure considers a special case in which the lobbying cutoff is higher than the export cutoff. Holding τ and η constant, Panels A and B plot firm lobbying expenditures and output distortions depending on their productivity levels. The x-axes are productivity ϕ.

Table B.1: Robustness. Estimation Results for the Parameter θ

Robustness	ETR		MRPK		Alternative Functional Form							
Dep.	$\ln 1 - \text{ETR}_{i,t+1}$		$\ln \frac{w_{nj,t+1} L_{i,t+1}}{K_{i,t+1}}$		$\ln 1/\text{TFPR}_{it}$		$\ln \text{Sale}_{it}$		$\ln 1/\text{TFPR}_{it}$		$\ln \text{Sale}_{it}$	
	OLS	IV	OLS	IV	OLS	IV	OLS	IV	OLS	IV	OLS	IV
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
$\Delta \ln Lobby_{it}$	−0.003	0.059**	0.002	−0.065								
	(0.003)	(0.028)	(0.003)	(0.054)								
$\Delta \mathbb{1}[Lobby_{it} > 0]$					−0.025	1.052***	0.528***	3.651***				
					(0.060)	(0.370)	(0.147)	(0.723)				
$\Delta\text{asinh}(Lobby_{it})$									−0.002	0.082***	0.048***	0.286***
									(0.005)	(0.030)	(0.012)	(0.055)
KP-F		12.46		12.46		14.86		14.86		12.59		12.59
AR		7.37		1.04		13.21		14.17		13.21		14.17
AR p-val		< 0.01		0.31		< 0.01		< 0.01		< 0.01		< 0.01
N	1,206	1,206	1,206	1,206	1,206	1,206	1,206	1,206	1,206	1,206	1,206	1,206

Notes. Standard errors are clustered at the state level. * p<0.1; ** p<0.05; *** p<0.01. This table reports the OLS and IV estimates of Equations (3.1) and (3.2). The dependent variables are the cash effective tax rates in columns 1-2, log wage bill divided by capital in columns 3-4, log inverse of TFPR in columns 5-6 and 9-10, and sales in columns 7-8 and 11-12, respectively. All specifications include corporate income tax, job creation tax credit, investment tax credit, R&D tax credit, property tax abatement, and transfers from the federal government, changes in state-industry wages, the initial lobbying status, and SIC 4-digit fixed effects. KP-F is the Kleibergen-Paap F-statistics. AR and AR p-val are the Anderson-Rubin test statistics and its p-value.

References

Hsieh, Chang Tai and Peter J. Klenow, "Misallocation and Manufacturing TFP in China and India," *Quarterly Journal of Economics*, 2009, *124* (4), 1403–1448.